Mathematical Ecology: Dynamical Systems of Spotted Owls and Blue Whales

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Abstract

In studying population like spotted owls and blue whales mathematical ecologists often pay close attention to the various numbers of a species at different stages of life. In this case study, we examine how eigenvectors and eigenvalues from Linear Algebra can be used to study the change in a population over time. Using real data from populations of spotted owls and blue whales, we determine the long-range population of the Northern spotted owl and whether the blue whale population is becoming extinct. The notion of a sustainable harvest is also introduced.

Introduction

In 1990, the Northern Spotted Owl created controversy in the Pacific Northwest – environmentalists were concerned because, as they claimed, the owl was threatened with extinction because of extensive old-growth forest logging in their habitats. Loggers, on the other hand, argued that the owl was not threatened and were concerned by the massive amounts of jobs to be lost if logging was prevented.

To handle the situation, mathematical ecologists renwed their drive and set out to better understand the population dynamics of the spotted owl. The life cycle of the spotted owl is divided into three stages: juvenile (up to a year old), subadult (1 to 2 years old), and adult (more than 2 years old). The owls mate for life after becoming subadults, live up to 20 years, and each pair requires about 4 square miles for it's own home territory.

Using a dynamical system to represent the owl population through their survival and procreation rates, these mathematical ecologists were able to predict the future populations of the owls.

The Model

The first step in studying population dynamics is to model the population at year intervals, where k = 0, 1, 2, … n. Also, it is assumed that for every female, there is one male (1:1 ratio), so only the female population is counted.

If there are j_k juvenile females, s_k subadult females, and a_k adult females at year k, then R. Lamberson et al. [2] found that the population of owls could be modeled by the equations followed by initial populations:

$j_{k+1} = 0.33a_k \quad j_0 = 40$
$s_{k+1} = 0.18\, j_k \quad s_0 = 20$
$a_{k+1} = 0.71\, s_k + 0.94\, a_k \quad a_0 = 140$

$$\longrightarrow \begin{bmatrix} j_{k+1} \\ s_{k+1} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & .33 \\ .18 & 0 & 0 \\ 0 & .71 & .94 \end{bmatrix} \begin{bmatrix} j_k \\ s_k \\ a_k \end{bmatrix}$$

The entries in the first row describe the **fecundity** of the population. Thus in the model above juveniles and subadults do not produce offspring, but each adult female produces (on the average) .33 juvenile females per year. The other entries in the matrix show **survival**. In this model, 18% of the juvenile females survive to become subadults, 71% of the subadults survive to become adults, and 94% of the adults survive each year. Note that the measures of fecundity and survival remain constant through time.

Material and Methods

We wish to determine the long-term dynamics of the spotted owl population given more recent data: whether the population is becoming extinct or is increasing.

Also, we wish to determine whether the population of blue whales is increasing or decreasing, and if it is stable, to determine the percentage of each class (of age) is in the stable population. Finally, we will estimate the what percent of the whale population can be harvested each year while keeping the population constant.

To answer these questions we will use Maple to examine the eigenvalues of the matrix A. If the corresponding eigenvectors are labeled v1, v2, and v3, the vector xk may be expressed as:

$$\mathbf{x}_k = c_1(\lambda_1)^k\mathbf{v}_1 + c_2(\lambda_2)^k\mathbf{v}_2 + c_3(\lambda_3)^k\mathbf{v}_3$$

Which is called the **eigenvector decomposition** of xk. We will also examine the population of juvenile, subadult, and adult females and the total population of spotted owls over the period of n years.

Based on these observations we will make some prediction regarding the future of the species: extinction or survival. If the population increases, we will find the long-term distribution of the owls by life stages.

We will do the same for the whale population, which is divided into the follow age classes: < 2 years, 2-3 years, 4-5 years, 6-7 years, 8-9 years, 10-11 years, 12 or more years.

Results

1. The long-term dynamics of the Northern spotted owl:

- The eigenvalues of the data matrix A are computed using Maple software: -.04 + .2i, -.04 - .2i, .94
- Because all of these values have a magnitude less than 1, as k increases, x^k tends towards the zero vector
- This set of data indicates that the population is becoming ***extinct***

$$\begin{bmatrix} 0 & .125 & .26 \\ .33 & 0 & 0 \\ 0 & .85 & .85 \end{bmatrix}$$

2. The long-term dynamics of the blue whale population:

- The eigenvalues of the data matrix are computed using Maple software: -.47, -.23 + .36i, -.23 - .36i, .16 + .58i, .16 - .58i, 1.01, .38.
- Because there is a value with a magnitude over 1, the population is increasing steadily at a rate of 1.01 (the only eigenvalue over one).
- This set of data shows that the population of whales would be ***increasing*** exponentially at a growth rate of 1.01; the population would be increasing by 1% per year.

$$\begin{bmatrix} 0 & 0 & .19 & .44 & .5 & .5 & .45 \\ .77 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .77 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .77 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .77 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .77 & .78 \end{bmatrix}$$

3. The percentage of each age class of blue whales in the stable population:

- The eigenvector corresponding to the eigenvalue that represents the population growth rate of 1.01 is computed to be: (.57, .43, .33, .25, .19, .15, .55)
- Dividing each element by the sum of the elements to get a percentage of the population for each age class → (.23, .18, .14, .10, .08, .06, .21)

4. Sustainable harvest of Blue Whale population:

- To estimate the percentage of the whale population that can be harvested while keeping the population constant, the following equation is solved:

$$\mathbf{x} = A\mathbf{x} - hA\mathbf{x} = (1 - h)A\mathbf{x}$$

where h is h is the percentage to be harvested for the is only eigenvalue greater than 1

- Given the data in Matrix A, the blue whale population to be harvested is: h = (1.01 – 1) / 1.01 = .0099. That is .99% of the population can be harvested while keeping the population constant.

Summary

- Linear algebra is a powerful mathematical tool that can be used in the form of dynamical systems to study the change in a population over time.
- Given a matrix where a the Northern spotted owl population is divided into age classes: the first row is the fecundity, and the rest shows the survival rate, it is possible to determine whether the population is increasing or decreasing over time by studying the eigenvalues of the matrix.
- By studying the corresponding eigenvector of an eigenvalue that determines the growth rate of an increasing population, it is possible to predict the long-term distribution of the population by life stages.
- In addition, it is possible to use eigenvalues to predict the percentage of a population that can be harvested while keeping the population constant.
- All of this helps us to keep nature in balance – whether it be saving owls from extinction or calculating the safe amount of whales to harvest each year.

References

1. David C. Lay, Linear Algebra and Its Applications, 3nd Edition (Pearson-Addison-Wesley, 2006)
2. Lamberson, R. H. et al. "A Dynamic Analysis of the Viability of the Northern Spotted Owl in a Fragmented Forest Environment." Conservation Biology 6(1992), 505-512.