Mathematical Ecology: Dynamical Systems of Spotted Owls and Blue Whales

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Abstract

In studying population like spotted owls and blue whales, mathematical ecologists often pay close attention to the various numbers of a species at different stages of life. In this case study, we examine how eigenvectors and eigenvalues from Linear Algebra can be used to study the change in a population over time. Using real data from populations of spotted owls and blue whales, we determine the long-range population of the Northern spotted owl and whether the blue whale population is becoming extinct. The notion of a sustainable harvest is also introduced.

The Model

The first step in studying population dynamics is to model the population at year intervals, where \( k = 0, 1, 2, \ldots, n \). Also, it is assumed that for every female, there is one male (1:1 ratio), so only the female population is counted. If there are \( j \) juvenile females, \( s \) subadult females, and \( a \) adult females at year \( k \), then R. Lamberson et al. [2] found that the population of owls could be modeled by the equations followed by initial populations:

\[
\begin{align*}
J_{k+1} &= 0.33 a_k + 40 \\
S_{k+1} &= 0.18 s_k + 20 \\
a_{k+1} &= 0.71 a_k + 0.04 a_k = 140
\end{align*}
\]

The entries in the first row describe the fecundity of the population. Thus in the model above juveniles and subadults do not produce offspring, but each adult female produces (on the average) 33 juvenile females per year. The other entries in the matrix show survival. In this model, 18% of the juvenile females survive to become subadults, 71% of the subadults survive to become adults, and 94% of the adults survive each year. Note that the measures of fecundity and survival remain constant through time.

Material and Methods

We wish to determine the long-term dynamics of the spotted owl population given more recent data: whether the population is becoming extinct or is increasing. Also, we wish to determine whether the population of blue whales is increasing or decreasing, and if it is stable, to determine the percentage of each class of age is in the stable population. Finally, we will estimate the weight percent of the whale population can be harvested each year while keeping the population constant.

To answer these questions we will use Maple to examine the eigenvalues of the matrix \( A \). If the corresponding eigenvectors are labeled \( v_1 \), \( v_2 \), and \( v_3 \), the vector \( x_k \) may be expressed as:

\[
x_k = c_1 (\lambda_1)^k v_1 + c_2 (\lambda_2)^k v_2 + c_3 (\lambda_3)^k v_3
\]

which is called the eigenvector decomposition of \( x_k \). We will also examine the population of females, subadult, and adult females and the total population of spotted owls over the period of \( n \) years.

Results

1. The long-term dynamics of the Northern spotted owl:

The eigenvalues of the data matrix \( A \) are computed using Maple software:

\[
\begin{array}{cccc}
-0.23 & 0.18 & 0.14 & 0.10 \\
0.36i & 0.94 - 0.23i & 0.71 - 0.36i & 0.04 + 0.2i \\
\end{array}
\]

\[
d = 0.04 + 0.2i, 0.94 - 0.23i, 0.71 - 0.36i, 0
\]

Because all of these values have a magnitude less than 1, as \( k \) increases, \( f \) tends towards the zero vector.

This set of data indicates that the population is becoming extinct.

2. The long-term dynamics of the blue whale population:

The eigenvalues of the data matrix are computed using Maple software:

\[
\begin{array}{cccc}
-0.47 & -0.23 & -0.36 & -0.36 \\
1.01 & 0.57 & 0.33 & 0.25 \\
\end{array}
\]

\[
d = -0.47, -0.23, -0.36, -0.36 \\
d = 1.01, 0.57, 0.33, 0.25
\]

Because there is a value with a magnitude over 1, the population is increasing steadily at a rate of 1.01 (the only eigenvalue over one).

This set of data shows that the population of whales would be increasing exponentially at a growth rate of 1.01; the population would be increasing by 1% per year.

3. The percentage of each age class of blue whales in the stable population:

The eigenvector corresponding to the eigenvalue that represents the population growth rate of 1.01 is computed to be:

\[
(0.57, 0.33, 0.25, 0.19, 0.15, 0.55)
\]

Scaling each element by the sum of the elements to get a vector with a length of 1.

4. Sustainable harvest of Blue Whale population:

To estimate the percentage of the whale population that can be harvested while keeping the population constant, the following equation is solved:

\[
\begin{align*}
0.57 x_0 &= 0.18 x_0 + x_1 \\
0.33 x_1 &= 0.71 x_1 + 0.04 x_1 \\
0.25 x_2 &= 0.04 x_2 + 0.04 x_2 \\
0.19 x_3 &= 0.04 x_3 + 0.2i x_3 \\
0.15 x_4 &= 0.04 x_4 + 0.2i x_4 \\
0.55 x_5 &= 0.04 x_5 + 0.2i x_5
\end{align*}
\]

where \( x_0 \) is the percentage to be harvested for the first age class, \( x_1 \) for the second, and so on.

Summary

• Linear algebra is a powerful mathematical tool that can be used in the form of dynamical systems to study the change in a population over time.

• Given a matrix where the Northern spotted owl population is divided into age classes: the first row is the fecundity, and the rest shows the survival rate, it is possible to determine whether the population is increasing or decreasing over time by studying the eigenvalues of the matrix.

• By studying the corresponding eigenvector of an eigenvalue that determines the growth rate of an increasing population, it is possible to predict the long-term distribution of the population by life stages.

• In addition, it is possible to use eigenvalues to predict the percentage of a population that can be harvested while keeping the population constant.

• All of this helps us to keep nature in balance — whether it be saving owls from extinction or calculating the safe amount of whales to harvest each year.

References