Mathematics in Nature: Bees, Honeycombs Bubbles, and Mud Cracks

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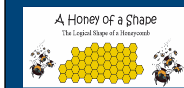
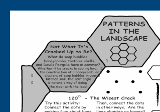


ABSTRACT

Polygons are fascinating, especially when they are approximated in nature. When looking carefully on can see them all around us. For example, hexagonal convection-cell clouds, mud cracks, or even sea salt flats exhibit the same pattern.
Here, we explore how polygons configuration is achieved throughout nature. We present three examples that exhibit patterns which can be described mathematically: honeycombs, bubbles and mud cracks. The hexagonal honeycombs maximize the enclosed region and minimize the wax needed for construction, while satisfying the bees' cell-size constraint. Bubbles are a real-world example of what mathematicians call a minimal surface. A minimal surface is one that has the smallest area possible, while meeting certain conditions. The drying mud becomes brittle and not only forms into its shapes of polygons but is separated by vertical cracks.



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Bees & Honeycombs

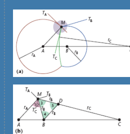
The most fascinating application of hexagons is the honeycomb. It is common knowledge that circular cells create wasted space, whereas hexagonal cells, such as the ones that can be found in honeycombs, are more efficient. In fact, if one places six regular hexagons in a ring with each pair sharing one common side, a free seventh hexagon appears in the center. This seven-hexagon honeycomb arrangement maximizes the area enclosed in the honeycomb while minimizing the perimeter. Which ultimately means the minimization of the wax needed for the construction of the arrangement. It is evident that bees do not have to take this course in optimization theory to make their honeycomb. This is because this same hexagonal pattern can be seen in a collection of identical coins placed together, in contact, on a flat surface. Although, by looking at this pattern of coins it becomes clear that if the bees wish to tessellate (covering a plane by using the repetition of a single shape with out gaps or overlap) a plane for their honeycomb a circle or cylindrical shape will not work. Some people believe, such as Pappus of Alexandria, that bees are gifted with "a certain geometrical forethought" and "There being, then, three figures which of themselves can fill up the space around a point, viz. The triangle, the square and the hexagon, and the bees have wisely selected for their structure that which contains more angles, suspecting that it could hold more honey than either of the two." At the same time, there are other, later, writers that believe that this hypothesis was incorrect, and that the hexagonal cell created by the bees was "no more than the necessary result of equal pressures, each bee striving to make its own little circle as large as possible." The honeycomb produced by bees is among works of art produced by animals. Their honeycomb is geometrically perfect and leads scientists to question whether it is the result of self-organizing principles of packing or genetic predisposition.

The regular tessellation Consisting of regular hexagons

Honeycomb Conjecture Theorem: Any partition of the plane into regions of equal area has perimeter at least that of the regular hexagonal grid (i.e., the honeycomb, illustrated above). Pappus refers to the problem in his fifth book. The conjecture was finally proven by Hales (1999, 2001).

Bubbles

Look virtually anywhere, and you can find bubbles. Whether they're from the soap in your kitchen sink or from the hot gasses vented from the ocean floor, all bubbles are subjected to the same mathematical rules and concepts. Bubbles are considered to be what are called, "minimal surface structures." Essentially, bubbles form a spherical shape that employs the least amount of surface area needed to enclose any given volume of air or gas. They remain spherical because there is an equal amount of pressure against all points on the bubble, therefore, no distinct edges can be maintained. The only exception to this rule is when multiple bubbles join together. Foam, for instance, is composed of many bubbles of various shapes and sizes. When they are joined together they change shapes to maximize their space-filling capabilities. Though their appearances can all be different, when three or more bubbles join together they merge so that the three angles of their connection form a 120 degree angle (the same number found in the hexagonal formation of bee hives). The bubbles alter their formations to minimize the energy required to contain the multiple volumes of gases, however, if all surrounding bubbles popped and only one remained, it would revert to its spherical shape.



(a) Two bubbles meeting at point M. All three bubbles meet at 120 angles-the three shaded angles are all 120 degrees. (b) The geometry associated with two bubbles. The pink angles are 30 degrees, and the green angles are 60 degrees.The numbers refer to the steps in the derivation in the text.

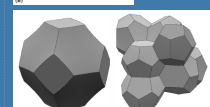
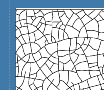


Figure 10 a (left): Kelvin's 1887 conjecture for the unit cell of the optimal tiling of space. This 14-faced cell looks much like the Wigner–Seitz cell of a bcc crystal, but its edges and faces are slightly curved. b (right): Weaire and Phelan's 1994 counterexample unit cell consisting of eight polyhedra (six 14-faced, two 12-faced). It is not known if this cell is indeed optimal.

Mud Cracks

As the Earth's surface loses water not only are mud cracks formed, but polygons as well. These polygons create a form of repeating symmetry throughout the dried mud puddles. As the mud loses water, this causes it to shrink and separate. The drying mud becomes brittle and not only forms into its shapes of polygons but is separated by vertical cracks. These polygons are made up of straight lines creating a path or circuit. This symmetry formed by mud puddles is also formed similarly by basalt and lava. Once they are dried or cooled down, polygons are formed leaving a symmetric pattern on the Earth's surface. This natural wonder is all around us. Without looking very far, symmetry can be found in just about anything.



source : Wolfram Mathworld

Theorem: The patterns of cracks observed in mud that has been dried by the sun form curves that often <u>intersect</u> in right angles.

Help! I'm cracking up!

Next time you bake a pumpkin pie, watch how it cracks as it cools. Do the cracks meet in a three-way conjunction? Do they form 120° angles?

Let's hear it for the hexagon! This shape is nature's way of using the least length of line to enclose the most area.

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