## Chaos in Ecology

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## ABSTRACT

Dynamical systems include mathematical models that are of great use dynamical systems that are highly sensitive to initial conditions. Chaotic behavior can be observed in many natural systems. The predator-prey mathematical population model can be used to analyze the effects one species has on another in a certain ecosystem, and also to determine any chaotic behavior in the species population. In
this study we examine actual data of Nile perch being introduced into Lake Victoria, their effects on the population of cichlids, and any chaotic behavior that occurs.


## INTRODUCTION

The Nile perch is an enormous fish native to the Ethiopian region. This fish was introduced to East Africa's Lake Victoria in an effort to boost the fisheries production of the lake. A dramatic increase in population size of the Nile perch coincided with a drastic decline or disappearance of many indigenous species.
Fisheries based on non-native or introduced fish species in many
aquatic environments have study aspects from species' establishment and performance in thei new habitats, to environmental impact, and socioeconomic benefits. The introduction of the Nile perch resulted in a fivefold increase in the amount of fish protein available to local communities. This consequently increased income and employment opportunities. Fish
catches from Lake Victoria made up more than $90 \%$ of the regions catches from Lake Victor
total fisheries production.
The tremendous increase in the Nile perch resulted in drastic changes
in the ecology of the lake. Prior to the introduction species thegy of the lake. Prior to the introduction of this non-native the Lake had a very diverse fish fauna with over 28 species. the ability to adapt to a wide range of conditions. With a high tevel adaptability and ecological versatility one would consider them to adaptability and ecological versatility one would consider the
have a level of protection against extinction. However, upon introduction of the Nile perch this cichlid fish population began to decline drastically. This is clear indication of the magnitude of change
that has taken place within Lake Victoria.

MATERIALS AND METHODS
The predator-prey system model describes the population growth of
two species. One species known as the prey provides sustenance for two species. One species known as the prey provides sustenance fo the other, the predator. This model allows one to analyze how a
population will either grow or decay exponentially. The linear prey predator model a relationship between two interacting species is as follows.
$P_{n+1}=r_{1} P_{n}-s_{1} Q_{n}$
$\mathrm{Q}_{n+1}=\mathrm{s}_{2} \mathrm{P}_{n}+\mathrm{r}_{2} \mathrm{Q}_{n}$
$P_{n}$ is representative of the prey and $Q_{n}$ of the predator. The $r$ value is defined as the intrinsic growth rate and all of the parameters are
non-negative. This prey-predator system is classified as linear for each of its two equations is a linear function of its two variables. If the system were to undergo immigration a constant, $k$, would be added to the equation. Similarly, if the population were to experience migration or harvesting, a constant would be subtracted from the equation. This system indicates the iteration of two equations for two interacting quantites $\mathrm{s}_{n}$ and Q $_{n}$ thuat evolve over
time matches the number of variables. In an effort to iterate the system the pair of initial conditions must be known.

We further implored the concept of initial condition sensitivity in relation to chaos. Solutions that begin close to one another may lead When dealing with a stable fixed point or periodic cycle the long term or asymptotic behavior of the set of solutions can be predicted. However, with a scrambled set there exist an infinite number of
different asymptotic behaviors Thus to determin the different asymptotic behaviors. Thus to determine the dynamics for any particular $X_{0}$ one must use direct numerical iteration. The true
initial $X_{0}$ can never really be known, implying long-term predictions in modeling are virtually impossible. With each step of the iteration any specific predictions become increasingly inaccurate. Thus the model is too sensitive to make practical long term predictions. This sensitive dependence is commonly referred to as the butterfly effect. For in
theory the motion of the butterfly's wings could have an effect on weather conditions, which we consider chaotic behavior.


## RESULTS

To see what results the system would yield an Excel file was created. cichlids were plotted against each other. This visual allows one to see clearly how the Nile perch population grew, while the cichlid population declined. Another graph was created in an effort to show the importance of initial condition sensitivity. Various initial conditions were plotted and tried to see how altering their values would change the outcome.

Pn vs. Qn (Qn initial = 2000)



Pn vs. $\mathrm{Qn}^{(Q n}$ initial $=8000$ )



## CONCLUSIONS

Studying this mathematical model we gained a deeper understandin eal-life ecological concern, we learned of the affects of adding just one non-native species to a Lake. The chaotic behavior and unpredictability found within ecology is truly fascinating! We encountered difficulties in finding real Nile perch data and realized thus how hard it is for scientist to predict the future of the lake. The situation in Lake Victoria appears to be a double edged sword.
"Some have decried the loss of Lake Victoria's native species, while others praise the introduction of the Nile perch (Kaufman)." Clearly the importance of biodiversity comes into question when a commodity of value appears in its place. There are many factors all experienced the appearance of high algae blooms associated with lower oxygen levels. However, it is important to realize that human intervention is not the only cause of a changing ecosystem. Shifting continents have the ability to shift the earth's flora and fauna. Ther re collisions with extraterrestrial debris and obviously naturally occurring ev
and disease.
The collapse of the species-rich ecosystem of Lake Victoria replaced by a simplified community raises numerous questions and concerns. The answers must address the dynamics of speciation extinction, the knowledge of non-native introductions and the practices of fisheries The predator-prey model can be advantageous to scientists analyzing consider. One such being mother nature, whose chaotic behavior one can never truly predict!

## REFERENCES

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